Research on the Application of the History of Mathematics in the Teaching of Higher Mathematics

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Abstract: Aiming at the problems of higher mathematics abstract, unique logic, difficult to understand mathematical thoughts and difficult teaching, this paper introduces the history of mathematics to show the formation process of higher mathematical thinking. According to the case teaching theory, combining the content of higher mathematics teaching and the content of mathematics history teaching, this paper puts forward an effective teaching method to infiltrate the case of mathematics history in teaching. Practice has proved that the infiltration of mathematics history knowledge in the teaching process has a positive effect on improving students' interest in learning mathematics, broadening their vision, and deepening their understanding of mathematical thinking.

1. Introduction

Advanced mathematics definition, theorem, description language is unique, logic abstract, it is difficult for students to understand the mathematical ideas it represents. Showing the history of the generation, improvement and final result of mathematical theorems can improve the level of understanding, increase the interest in learning, and solve the problems of higher mathematics teaching. It is necessary to study the application of mathematical history knowledge in higher mathematics teaching.

The role of the history of mathematics in mathematics teaching has been attached great importance in the world, and a special international research organization HPM has been established to carry out long-term research[1]. Without understanding the history of mathematics, one cannot fully understand the subject of mathematics [2,3]. Wang Qingjian elaborated the current situation and existing problems of the history of mathematics education in China, and put forward suggestions for the reform of the history of mathematics education in a systematic, scientific and modern way [4].The purpose of mathematics history education is to better understand mathematics, stimulate the interest in learning mathematics, and cultivate humanistic quality [5]. The Process of Calculus reveals the necessary connection between the tortuous process and the final result[6]. Great progress has been made in related research in China, but further research is needed in application.

This paper analyzes the influence of the history of mathematics education on the teaching of higher mathematics, explores the main factors that affect the effective implementation of the history of mathematics education in colleges and universities, and carries out the practice and exploration of the history of mathematics teaching, which improves the teaching quality and has a reference for the popularization and promotion of the history of mathematics teaching.

2. Higher mathematics teaching problems

2.1. Theorem definition language is difficult to understand

There are many concepts and theorems in higher mathematics, the description language is unique, the logic is abstract, it is difficult for students to understand the true meaning of concepts and

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theorems, and they can not master the ideas of higher mathematics. For example, the definition of limits.

Definition: Let the function y=f(x), if for any given positive number (no matter how small), there is always a positive number N, such $|x - x_0| < \delta$ that for all x that fits the inequality, the corresponding function value f(x) satisfies the inequality

$$|f(x) - A| < \varepsilon$$

Then the constant A is called the limit of the function y=f(x) as $x \to x_0$, denoted as:

$$\lim_{x \to x_0} f(x) = A$$

This is a function limit concept described in language, which is difficult for students to understand. Why portray it that way? If it is defined as follows: the independent variable x is in the process of forever changing near x0, the function value y gradually approximates some certain value A, and "can never coincide with A", it may be easier to understand, but it is not an exact mathematical expression. It may be helpful for students to understand if they can explain the process of producing this language, such as the corresponding process of the history of mathematics.

2.2. It is difficult to connect elementary mathematics teaching with higher mathematics

Due to the new round of curriculum reform, higher mathematics is taught in middle schools, which increases the content of middle school mathematics textbooks. However, in fact, the teaching system of universities and middle schools is independent, and the teaching content is not reasonably distributed, and there are many repetitions, disconnects and cohesion problems.

Due to the influence of exam-oriented education, the teaching method of middle school is mainly instilled, the speed is slow, the theoretical depth is shallow, the focus of the teacher is to do the problem method, and in the university, the focus of the teacher is the theory, the teacher is often the outline of the teaching style, the teaching speed is fast, the abstractness is strong, and the communication with the students after class is also less. Students are required to have a certain degree of independence and autonomy in learning, from the past passive follower learning to active and conscious learning, teaching methods and learning methods can not smoothly transition.

Mathematics education, the education of mathematics thought is not enough attention. The point-to-point function thinking of elementary mathematics is different from the limit thinking of higher mathematics, and the production process of these two kinds of thinking is explained in the history of mathematics.

2.3. Lack of connection between theorems

There are many concepts and theorems in advanced mathematics, which form a strict logical system, and the derivation process is complex, which is difficult for students to understand, and the logical connection between theorems is even more difficult to understand, which becomes a learning disability. Students must have higher mathematical literacy and thinking ability to understand the mathematical ideas expressed in the entire logical system.

Now teaching the content of separate theorems and the proof of theorems, only pay attention to a single theorem, out of the internal connection of higher mathematics thought system, students can not understand, only rote memorization. You need to understand why this theorem is needed, why it is described this way. The history of mathematics describes the process of the formation and completion of mathematical thinking, and illustrates by example that it would not be possible otherwise. The story of the history of mathematics can let students know that the logic of theorems does not come easily, and the elaboration of theorems solves many problems, which helps students understand the contents of theorems and the role of higher number systems.

Mathematics is a kind of philosophical thinking, only by mastering the thinking method can we find the application scenario in real life and dispel the negative attitude towards mathematics curriculum.

3. Permeate the importance of the history of mathematics

3.1. Understand the past and future of higher mathematics

Most students think that advanced mathematics is abstract, the mathematical language is difficult to understand, the theory is profound, and there is no use in work. In fact, higher mathematics is widely used in military, economy, management, finance, medicine and other fields, and is closely related to people's lives. Practical and problem solving is the eternal theme of mathematics, which needs to be analyzed from the perspective of mathematics history to show the origin, development and function of higher mathematics content.

For the concepts and theories in higher mathematics, if you can understand its context, you will have a deeper understanding. The history of mathematics shows the development of mathematics, can comprehend the essential law of mathematics, and predict the future process of mathematics. If a mathematical problem is solved, the solution is called a formula or theorem; if the problem is temporarily unsolvable, it is called a conjecture. Higher mathematics is composed of many theorems and formulas. If the history of mathematics is infiltrated into the teaching process of higher mathematics, students can understand the origin and development of formulas and theorems in higher mathematics. It has a better effect than simply explaining the content of definitions and theorems.

Problem is the core of mathematics, the source of concepts and theorems and the reason for existence. Mathematics should be given a higher status in teaching. There are many important problems in higher mathematics, such as calculus, limits, and analytic geometry. In the process of solving these problems, mathematicians put forward new mathematical theorems and created new branches of mathematics, and made a lot of efforts to prove, rigor and complete these mathematical thinking, and finally presented the theorems that appeared in the end. Two conclusions can be drawn, one is that mathematics exists in order to solve problems, the other is that the method of solving problems needs long-term efforts to improve, before becoming the current mathematical theory. Only by infiltrating the history of mathematics in the teaching process of higher mathematics and showing the process of mathematical problems and solutions can students really understand these contents. When encountering problems, we can creatively apply mathematical thinking to solve problems.

3.2. Mathematical superstars light up our future

From the point of view of the history of mathematics, higher mathematics has experienced a long time from its emergence to its development. Mathematicians' unremitting efforts, brilliant achievements and supreme honors are enough to stimulate students' interest in learning higher mathematics and establish their confidence in learning higher mathematics well.

There is no shortage of genius in the field of mathematics. Like Archimedes, he discovered the parabola and parallel strings. And Pythagoras, he invented the very useful Pythagorean theorem. And then there's Fourier, who created the famous Fourier formula.

Euler, one of the most prominent figures in the field of mathematics in the 18th century and the most prolific mathematician in the history of mathematics, wrote an average of more than 800 pages of papers per year, and also wrote a large number of textbooks on mechanics, analysis, geometry, variational method, etc., and most of them have become classics in the field of mathematics. In mathematical theory, we often see important constants, formulas and theorems named after Euler. Euler extended the whole of mathematics to physics, architecture, ballistics, navigation, and so on.

Euler is known as the "king of mathematics", the genius of geniuses, created the God formula, is called God. By the time he was ten years old, Euler was well ahead of his teachers in mathematics. By the age of 13, he had completed all his secondary school courses, and then entered the University of Basel, where he completed six professional degrees in just two years. At the age of 15, Euler graduated from the University of Basel, where he left behind a legend. In just one year, Euler completed the coursework required for his master's degree. Euler wrote 400 works on mathematics. Euler's life was a life of glory.

Newton's main mathematical achievements: 1. Calculus (fluid number method), 2. Binomial

theorem, 3. The center of curvature and curve circle are introduced in analytic geometry, and the calculation formula of curvature and the method of calculating curve curvature are presented. In addition, there are elementary number theory, numerical analysis and so on. 4. Invented the polar coordinate system. He was the first natural scientist in human history to receive a state funeral, and his burial place is called Westminster Abbey, which is the third institution of higher learning after Oxford and Cambridge. At the same time, it is also an important place for British celebrations, where celebrities can be buried here after death as an honor. The inscription on his tombstone reads: Let people hail what a great human glory has ever existed in the world.

3.3. Mathematics research spirit good material

The development of mathematics is not smooth sailing, most of the mathematical achievements are explored through a tortuous and difficult process, and the history of mathematics also records the fact that mathematicians study hard, rigorous innovation and devote themselves to the cause of science.

In the process of higher mathematics teaching, we should not only learn the basic knowledge and basic skills of higher mathematics, but also learn the creative spirit of mathematicians and the perseverance of assiduously studying. It is necessary to exercise students' will to overcome difficulties, form an attitude of seeking truth from facts and the habit of questioning and independent thinking. We should understand the basic ideas of mathematics, accumulate the basic experience of mathematics, and cultivate students' curiosity and thirst for knowledge in higher mathematics. Infiltration of some mathematical history content in higher mathematics teaching will make students feel passionate and confident in higher mathematics learning.

In the process of higher mathematics teaching, when it comes to Euler's formula, if the teacher can tell Euler's life to the students, the students will be touched. Euler was hailed as a "mathematical hero" and a model of struggle against fate. He left the motherland at the age of 20 to drift abroad, did not give up the Swiss nationality in his life, and the research data was burned up by the fire, but all this did not destroy his will, he still devoted himself to the cause of mathematics, almost in the main branches of mathematics will see the mathematical theorem or formula named after Euler, becoming the most fruitful mathematician in history. Then there is the mathematician Hua Luogeng, who is well known to us. He suffered from typhoid fever when he was 18 years old. Although he recovered from the disease, his left leg was disabled. Hua Luogeng in the left leg disability, walking is very inconvenient, when the left leg first draw a big circle, the right foot and then take a small step, he vowed to use a sound mind instead of sound legs, he is relying on self-taught mathematicians, he became a junior high school education to the United States, Germany and other multinational Academy of Sciences academician. His famous saying is: "Wisdom lies in learning, genius lies in accumulation.".

4. The difficult course of the development of calculus

4.1. The creation of calculus

The idea of calculus originated in ancient times, and encountered problems: trying to solve the problem of calculating the area of a circle and the tangent of the ball with some segmentation strategy. How to segment and to what extent, later rose to the elusive question of "infinitesimal" quantities and "limiting" processes.

Newton and Leibniz built on the work of their predecessors to create the method of differentiation and integration. The idea of limits is the soul of calculus. In philosophy, the finite and the infinite represent the dialectical nature of the motion of matter in time and space, and are opposites. In mathematics, finite represents a number, while infinite represents a process, the two are fundamentally different, but they are unified in the idea of limits. Through the bridge of limit, finite and infinite can be transformed into each other.

4.2. Development of calculus

If Newton and Leibniz were the architects of calculus, it was the extensive work done by Jacob Bernoulli (1654-1705) and John Bernoulli (1667-1748) that established calculus into the discipline it is known today. They read Leibniz's papers from 1684 to 1686, grasped the essence, fleshed. and then, through communication with Leibniz and with each other, perfected the unity and order of calculus. The word "integral" was given by Jacob, [2] and in their hands calculus became something that is readily accepted by today's students, with basic derivation development, calculus and the solution of elementary differential equations.

By 1956, the Complete Works of Euler had been published in more than 70 volumes and reached more than 25,000 pages, which fully proved Euler's exceptional talent. This talent is particularly evident in analysis, which includes landmark texts on functions, differential calculus, and integrals, as well as dozens of papers on topics ranging from differential equations to infinite series to elliptic integrals. Whether it is differentiation or integration, whether it is approximation or interpolation, it shows amazing originality. Von Neumann called Euler "the most outstanding mathematician of his time" because he asked many of the right questions and often found the right answers with his amazing mind and intuitive thinking abilities. [3] Euler was called "the embodiment of analysis" by later generations.

4.3. Calculus rigidification

At the end of the 18th century, the logical crisis of calculus remained unresolved. The work of D 'Alembert and Lagrange and other mathematicians working on these issues did not quell the tide of criticism. At the beginning of the 19th century, analysis went beyond what its predecessors could imagine and became a subject of universality, abstraction and inequality.

(1) Cauchy and the basis of analysis

Cauchy, the French mathematician who pioneered analytical rigor in the 19th century. He gave clear definitions of the basic concepts of calculus, such as limits, variables, functions, continuity, differentiation, derivatives, convergence, etc. Cauchy's work was a crucial step towards the overall rigor of analysis. Many of his definitions and statements are already quite close to the modern form of calculus.

(2) Riemann

Until the early 19th century, the "legality" of plural numbers remained an unresolved issue. But complex analysis really became a field of study in modern analysis, established in the 19th century, mainly by Cauchy, Riemann, and Weierstrass, Riemann received his doctorate at the University of Gottingen with his paper "The Basis of Functions of Complex Variables" (1851).

(3) Weierstrass

Weierstrass also blazed a trail for complex functions, and his work has always been known for its rigor. He believed that the principles of function theory must be based on algebraic truths, so he set his sights on power series, using them to define the analytic properties of functions in the domain of one point, and to derive the entire analytic function. A power series is used to represent complex functions that have been given in analytic form [5].

Weierstrass introduced rigorous arguments into analysis, established the theory of real numbers, defined irrational numbers in terms of increasing bounded sequences, created a set of e- δ language, and gave the upper and lower limits of series. Weierstrass obtained the concept of uniform convergence in about 1842, and used it to give the conditions for term-by-term integration of series and differentiation under the integral sign. Weierstrass's contributions to analytical rigor earned him the title of "Father of modern analysis".

5. The history of mathematics into the methods of higher mathematics

5.1. Strengthen the understanding of the history of mathematics

At present, the teaching of higher mathematics can only inculcate content, and the knowledge of the history of mathematics is rarely applied. It is not easy to explain why mathematical axioms are

produced, why they are expressed in this way, and why they are precise and accurate.

The essence of the development of mathematics is the progress and development of people's mathematical thoughts, which is the concentrated embodiment of the creative work of each mathematician. The achievements of mathematicians' work, the charm of mathematicians' personality and the pioneering ideas of mathematicians are all reflected in the development of mathematics. The development of mathematics is not isolated, the development of mathematics drives the social progress, and the social development in turn affects the direction and results of mathematical research. All of this is reflected in books on the history of mathematics, including the history of mathematics with a chronological trace.

Understanding the history of mathematics enables students to have a certain understanding of the generation and evolution of mathematical ideas in the process of problem generation, development and final problem solving, humanizing mathematics and enabling students to truly grasp the essence of mathematics.

5.2. Use the history of mathematics to explain mathematical content

(1) Use the history of mathematics to explain the tightness of the expression

A mathematical problem, the process of solving a mathematical problem, the important mathematical ideas and methods, and the important conclusions obtained can explain the rigor of the relevant theorems. For example, how to understand the definition of limits:

To learn the development history of calculus well, stimulate students' interest in learning, and let students understand that it is not terrible, it is the top priority to solve the difficult problems in the process of learning higher mathematics.

Definition: Let the function y=f(x), if for any given positive number (no matter how small), there is always a positive number N, such $|x - x_0| < \delta$ that for all x that fits the inequality, the corresponding function value f(x) satisfies the inequality

$$|f(x) - A| < \varepsilon$$

Then the constant A is called the limit of the function y=f(x) as $x \to x_0$, denoted as:

$$\lim_{x \to x_0} f(x) = A$$

We encourage you to use your own understanding, your own language, to redescribe the definition of limits, you have the following descriptions:

The value of X is near x0, and the value of Y is near the number A.

The value of X changes around x0, and the value of Y changes around the value A.

The value of X is in the neighborhood of x0, and the value of Y is in the neighborhood of the number A.

The interval for change in X corresponds to the interval for change in Y.

All of the above definitions have correct mathematical meanings to some extent, but they are not complete and rigorous mathematical definitions. In fact, there are many similar definitions in the history of mathematics, which have been overturned by refutations. Only those that can withstand rebuttal can remain until now. After two thousand years of evolution, mathematicians such as Bernoulli, Cauchy, and Weierstrass redefined limits using inequalities, to the present-day textbook descriptions of inequalities. [6] Therefore, when answering the definition of limit, using the definition in the textbook is the most appropriate and rigorous answer.

(2) Open elective courses well

In recent years, some colleges and universities have set up elective courses on the history of mathematics. Although there are some shortcomings in the content and teaching level of the course, they are welcomed by students and play a positive role in the growth of students. In order to solve the problem of material selection, the topic of mathematics history related to mathematics teaching is selected to improve the efficiency.

6. Conclusion

The role of the history of mathematics in mathematics education has gradually become a hot spot in the research of mathematics education in our country. The history of mathematics is an inexhaustible and inexhaustible spring of mathematics life, which has immeasurable educational value.

The education of the history of mathematics in the teaching of higher mathematics can show the innovative thinking of mathematicians to solve problems, the pursuit of perfection and the never-ending spirit of exploration by using higher mathematics, which is helpful to improve students' interest in learning mathematics, expand their horizons and encourage them to learn mathematics well.

The addition of "mathematical historical materials" in the teaching materials and the infiltration of mathematical history in the teaching process will play a very positive role in explaining the correctness, rationality, rigor and coherence of the teaching content, and will contribute to a deep understanding of the teaching content of higher mathematics.

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References

[1] Li Wenlin. Introduction to the history of Mathematics. Beijing: Higher Education Press, 2008:1-9, 155-165,247-251.

[2] P R Halmos. Translated by Tang Shengchang. The core of mathematics. World Science, 1982, 7:47-49.

[3] Wang Qingjian. The history of mathematics and the reform of mathematics education. Journal of Mathematics Education.1995, 4:64-67.

[4] Wang Qingjian. Brief History of Mathematics. Beijing: Science Press, 2004:193

[5] By William Dunham. Translated by Li Bomin, Wang Jun and Zhang Huaiyong. The history of calculus. Shanghai: Shanghai Science and Technology Education Press, 2010:11-52.

[6] KATZV.Using History to Teach Mathematics.an International Perspective Washington: Mathematical Association of America.2000.